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SHIP'S SPEED-MEASUREMENT UNCERTAINTIES CAUSED BY LOCALIZED WATER MOVEMENTS

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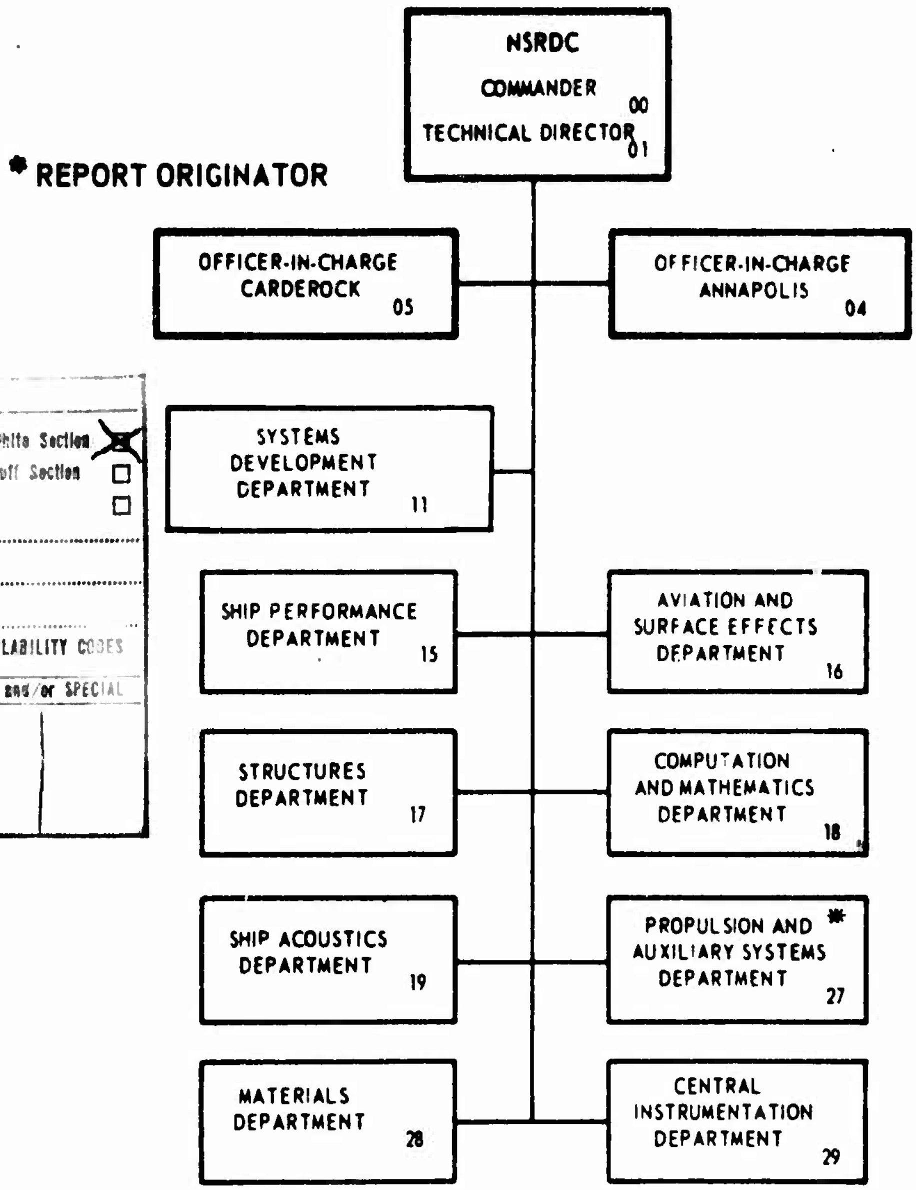

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ADMINISTRATIVE INFORMATION

This work was performed under the supervision of Mr. L. W. Griswold, Head, Sensing Systems Branch, as part of Work Unit 1-2732-101, Development of Improved Ship's Speed Sensors (funded under Project F 43 423, Task Area SF 43 423).

ACKNOWLEDGMENTS

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REPORT HIGHLIGHTS

OBJECTIVE

The objective of this study is to analyze the effects of local water motion on the useful accuracy of sensors that measure ship's speed relative to water. The water motions considered arise from surface waves, internal waves, turbulence, and vertical gradients of horizontal currents. The oscillatory and offset errors in speed signals affect fire-control solutions and navigation.

APPROACH

The report: (a) describes the phenomena with mathematical relations between local water motion (such as wave orbital velocities) and environmental parameters (such as wind), (b) estimates the types and magnitudes of speed sensing errors induced by the environments, and (c) presents ranges of errors a naval vessel could expect.

RESULTS

The study identifies several problem areas in speed sensing:

- Surface waves cause oscillating horizontal currents with peak magnitudes as high as several knots. The magnitudes decrease with depth, with the spectral peaks tending toward lower frequencies (or longer wavelengths) at increasing depths. This spectral change may cause difficulties for submarines in selecting a data sampling and processing or averaging scheme that can reduce wave effects and still respond to ship maneuvers.

- The horizontal components of oscillating currents associated with internal waves are proportional to the wave amplitude and phase speed and have magnitude up to tenths of knots. Since the wave amplitudes do not exceed the upper layer thickness (depth to thermocline) and the wave phase speed can be calculated or measured, it should be possible to estimate the upper limit to the horizontal currents associated with internal waves by using local measurements.

- For vessels of larger mass in relatively rapid local water movements, the water motion causes fluctuations in the indicated water speed while the speed over ground is essentially constant. For vessels of lesser mass in slowly varying local water movements, the speed relative to water is very nearly constant while the speed over ground varies as the water motion. These results follow directly from the inertial and drag characteristics of the vessel.

- Turbulence causes random but small (~0.01 to 0.1 knot) fluctuations in the speed readout. The spectral form appropriate to homogeneous, isotropic turbulence may be useful in predicting speed readout variations.

- Local ocean currents tend to decrease in magnitude and shift directions as a function of depth. A submersible and a surface support vessel operating together will almost certainly experience differences between the indicated water speeds of the two vessels even while they maintain zero relative velocity. Typical gradients for the North Atlantic show about 0.4-knot change from the surface to a depth of 50 meters.

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NOTATION

A - Empirical constant in equation (5), $A = 8.29 \times 10^{-7}$

a - Wave amplitude, equation (8)

B - Empirical constant in equation (5), $B = 4.76 \times 10^{-4}$

c - Internal wave speed defined by equation (7)

C_d - Drag coefficient, equation (13)

D(r) - Velocity structure function, equation (19)

d - Depth of water

E(k) - Turbulence spectra

f - Frequency

g - Acceleration of gravity

h' - Mean depth of upper layer (see figure 4)

h'' - Mean depth of lower layer (see figure 4)

k - Wave number ($k = 2\pi/\lambda$, where λ = wavelength)

M - Mass of submarine

m - Number defined by equation (2) or (3).

N - Brunt-Väisälä frequency, equation (10)

r - Separation distance, equation (19)

S(f) - Surface wave spectrum, defined by equation (5).

T - Thrust of submarine propeller.

T_o - Mean thrust

t - Time

U - Horizontal current caused by internal waves, equation (9)

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$\overline{u^2}$ - Subsurface velocity variance caused by wind waves, equation (1)

V - Speed of submarine relative to ground

V_o - Mean speed of submarine relative to ground.

V_I - Indicated speed relative to water

ΔV - Small perturbation from mean speed

v - Instantaneous current, equation (12)

v_o - Maximum amplitude of current, equation (12)

W - Wind speed

z - Vertical coordinate

ϵ - Energy dissipation rate

η - Density discontinuity surface, (figure 4)

θ - Latitude

κ - Turbulence wave number

ρ - Density

ρ' - Density of upper layer (see figure 4)

ρ'' - Density of lower layer (see figure 4)

σ - Frequency of internal wave, equation (8)

ϕ - Phase angle defined by equation (16)

ω - Radian frequency seen by observer, equation (12)

INTRODUCTION

Naval vehicles require a self-contained speed-measurement capability for inputs to systems such as the ship's inertial navigator or the fire-control computers as well as for navigation when external position fixes are not available. Most present day vehicles have log systems that measure the speed of the ship relative to the water. Thus, motion of the local water in the sensing region will cause a variance (error) in the speed signal.

The purpose of this study is to analyze the effects of local water motion on the useful accuracy of sensors that measure ship's speed relative to water. Four factors are considered. Three are: surface waves, internal waves, and turbulence, all of which cause oscillatory or random errors. The fourth factor is vertical current gradients that cause errors depending on the depth of sensing region. These factors are considered local in the sense that they occur over distance scales from tens of feet to several miles and time scales of minutes to several hours.

The basic theory describing the water motions is given in the summary by Williams[1] with example data taken from standard oceanographic atlases.[2,3]

UNCERTAINTIES CAUSED BY SURFACE WAVES

The water particles below a passing surface wave undergo an elliptic motion with the size of the ellipse decreasing exponentially with depth.[4] Ideally, the elliptic path closes on itself and no net movement of the water occurs. In reality, nonlinearties of the wave motion exist, and there is a small net transport current in the direction of the wave movement. However, even with little or no net transport, the oscillatory motion of the water will introduce a fluctuating velocity signal with a peak amplitude that may be as high as several knots.[4] With wave periods as long as 20 seconds, these velocity fluctuations may cause appreciable errors in computations based on water speed. When the vessel is moving in the direction of the most significant wave field, the doppler shift may cause the apparent period to be much longer with the attendant difficulty of averaging out the wave-induced water velocities.

The quasi-periodic oscillations in the indicated speed become very significant when the speed data are sampled at some relatively low frequency for input into a computer. If the sampling period is harmonic with one of the dominant wave periods, the computer averaging may be incorrect. The sampling problem is complicated for submarines because the dominant wave period increases with depth. A sampling frequency which is optimum at one depth is not necessarily optimum at another. A compensating factor, however, is the exponential decrease of the wave currents with depth.

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[1]Superscripts refer to similarly numbered entries in the Technical References at the end of the text.

The variance of the subsurface velocities caused by surface waves is calculated as follows.[5] Assume the horizontal velocity components have a normal distribution with a zero mean. The normal distribution is completely specified by its variance. From hydrodynamic potential theory the subsurface velocity variance $\overline{u^2}$ is related to the surface wave spectrum S(f) by

$$\overline{u^2} = \int_0^\infty \left[\frac{2\pi f \cosh m(d+z)}{\sinh md}\right]^2 S(f)\, df \tag{1}$$

where m is defined by

$$(2\pi f)^2 = mg \tanh md \tag{2}$$

and

f = frequency of surface or wave spectra, Hz*

d = depth of water to bottom

z = distance from surface to point $\overline{u^2}$ is considered ($z \leq 0$)

g = acceleration of gravity.

If the water depth is much greater than the longest wavelength considered, then tanh md is very nearly unity and we may write

$$m = \frac{4\pi^2 f^2}{g} . \tag{3}$$

Using the approximations for deep water where $d \gg \lambda$, the expression for $\overline{u^2}$ is simplified to

$$\overline{u^2} = \int_0^\infty 4\pi^2 f^2 \exp\left(\frac{8\pi^2 f^2}{g} z\right) S(f)\, df. \tag{4}$$

The surface or wave spectra can be described by the Pierson-Moskowitz spectrum for a fully developed sea.[6,7]

$$S(f)\, df = Ag^2 f^{-5} \exp\left[-B(g/fW)^4\right] df \tag{5}$$

*Abbreviations used in this text are from the GPO Style Manual, 1973, Unless otherwise noted.

where:

$A = 8.29 \times 10^{-7}$

$B = 4.76 \times 10^{-4}$

g = acceleration of gravity

W = wind velocity

f = frequency.

The variance of the horizontal component of the subsurface velocities caused by surface waves is related to wind velocity and depth by the integral equation:

$$\overline{u^2} = \int_0^{\infty} 4\pi^2 A g^2 \exp\left[(8\pi^2 f^2 z/g) - (Bg^4/f^4 W^4)\right] f^{-3}\, df. \qquad (6)$$

Figure 1 shows sections of analog records of the variations in the speed signal from an electromagnetic speed sensor on a submarine at various depths beneath a state 6 sea. The curves shown in figure 2 represent calculations based on equation (6). The circled points were obtained from the submarine data.

UNCERTAINTIES CAUSED BY INTERNAL WAVES

The gravity waves observed on the sea surface are a special example of the more general case of waves that may exist wherever there is a vertical density gradient. Such a density gradient exists at all times throughout an ocean-water column, but it is greatest in the thermocline. The density gradient associated with a thermocline is illustrated by the two examples taken from standard atlases[3] and shown in figure 3.

To simplify the mathematics, consider a two-layer model where the density gradient is a step change between the layers (see figure 4). For a shallow upper layer and deep lower layer ($k\,h''$ large, $k\,h'$ small), the wave speed is given by

$$C \approx \left[gh'\left(\frac{\rho''-\rho'}{\rho''}\right)\right]^{1/2} \qquad (7)$$

where

c = wave phase velocity

g = gravity

h' = thickness of upper layer

ρ' = density of upper layer

ρ'' = density of lower layer.

Assuming the vertical displacement of the interface is a simple cosine,

$$\eta = a \cos (kx - \sigma t), \tag{8}$$

where:

a = wave amplitude

k = wave number ($k = 2\pi/\lambda$)

σ = frequency;

the horizontal flow in the upper layer is given by

$$U = -\frac{a}{h'} C \cos(kx - \sigma t). \tag{9}$$

The frequency of the possible natural internal waves is bounded by the Brunt-Väisälä or stability frequency on the highest end and the inertial frequency on the low-frequency side. These frequencies are

$$N \simeq \left[-\frac{g}{\rho}\frac{d\rho}{dz}\right]^{1/2} \quad \text{[Brunt-Väisälä frequency]} \tag{10}$$

where

g = gravity

ρ = density

$\frac{d\rho}{dz}$ = mean density gradient

and

$$f_{inertial} = \frac{\sin \theta}{12 \text{ hours}} \quad \text{[inertial frequency]} \tag{11}$$

where θ = latitude.

The frequency and propagation velocity of the internal wave are determined to a large extent by the density gradient. However, the wave amplitude (hence, horizontal currents) are determined primarily by the generating source. The induced currents are directly proportional to the ratio of wave amplitude to the upper layer thickness as shown in equation (9). Since the amplitudes of the internal waves in the ocean do not exceed the upper layer thickness ($a < h'$), the wave-induced currents will be less than the phase velocity of the wave,

$$U < c.$$

Representative current magnitudes attributed to internal waves are of the order of 1 to 20 cm/s.[8]

The effect of the oscillating horizontal current of an internal wave on submarine motion is derived in appendix A. The drag forces of a submarine depend on its velocity relative to the water whereas the acceleration forces depend on changes in speed relative to ground. The oscillating currents of internal waves affect both speed over ground and speed relative to water.

Consider an oscillating horizontal current component v, seen by an observer, being translated at steady speed V_o. Thus,

$$v = v_o \sin \omega t \tag{12}$$

where

v = instantaneous current seen by observer

v_o = Maximum amplitude of current

ω = radian frequency seen by observer

t = time.

A submarine of mass M moving through this flow at constant thrust T_o will undergo accelerations because of drag variations from changing flow. Let V_o be the submarine water speed corresponding to $T = T_o$ with $v_o = 0$. Assuming the drag is proportional to the square of the velocity through the water, we have

$$M \frac{dV}{dt} + C_d(V-v)^2 + T_o = 0 \tag{13}$$

where:

V = speed of submarine relative to ground

C_d = drag coefficient

M = mass of submarine

T_o = constant propeller thrust.

By expressing the velocity of the submarine over the ground as

$$V = V_o + \Delta V \tag{14}$$

(where ΔV is a small perturbation from the average speed V_o) and assuming that

$$V_o >> |v - \Delta V|$$

the solution to equation (13) is found in appendix A to be

$$V = V_o + v_o \cos \varphi \sin(\omega t + \varphi) \tag{15}$$

where

$$\tan \varphi = -\frac{\omega M}{2 V_o C_d} \quad . \tag{16}$$

The indicated speed, V_I (or speed relative to the water) is just V-v or

$$V_I = V_o + v_o \sin \varphi \cos(\omega t + \varphi) \quad . \tag{17}$$

Using these equations and the known mass and drag characteristics of the submarine, the effects of current fluctuations on speed sensing can be determined. First, the differences between V_o and both V and V_I are proportional to the peak current magnitude v_o. Second, both V and V_I depend on the angle φ, which is related to the submersible and current parameters through equation (16). As $\varphi \to o$, $V_I \to V_o$ and $V \to V_o + v_o \sin \omega t$. The indicated speed approaches the constant valve V_o, but the speed over the ground varies as the oscillatory current. This case represents a small submersible with a relatively high drag in a slowly changing current. The other extreme, as $\varphi \to 90°$, results in $V_I \to V_o - v_o \sin \omega t$ and $V \to V_o$. This case represents a large submersible with low drag in a rapidly varying current.

The results of an analog computer solution of equation (13) are given in figure 5 for the assumed parameters:

M = 5,000 tons

V_o = 5 kn

T_o = 22,000 lb

$v_o = 0.5$ kn*

$\omega = 0.0105\ s^{-1}$ (for item (a), figure 5, a 10-min wave period)
$= 0.00105\ s^{-1}$ (for item (b), figure 5, a 100-min wave period)

UNCERTAINTIES CAUSED BY TURBULENCE

Turbulence is an irregular fluctuation of flow superimposed on the general or mean flow pattern. Small-scale turbulence covering a few tens of meters will generate irregular fluctuations in the speed measurement. While these small-scale turbulent eddies cause speed fluctuations that tend to average to zero, there will be an instantaneous error with a nonzero variance.

The ocean is neither homogeneous nor isotropic, yet measured velocity turbulence spectra at the large wave numbers tends to follow the power law relation that is applicable to homogeneous and isotropic turbulence.[9,10]

$$E(\kappa) = 1.44\ \epsilon^{2/3}\ \kappa^{-5/3} \tag{18}$$

where ϵ is the kinetic energy dissipation rate and κ is the wave number of the turbulent eddies. From measured data, the $\kappa^{-5/3}$ seems to hold for a given depth for frequencies as small as 1 cycle per 10 hours.

Assuming, for a given depth, that the horizontal velocity components satisfy the conditions for equation (18), then a velocity structure function, D(r), can be defined

$$D(r) = \overline{[V(r_1) - V(r_2)]^2} \tag{19}$$

where $V(r_1)$ and $V(r_2)$ are the instantaneous velocity components at positions r_1 and r_2 and where $r = |r_1 - r_2|$. The velocity structure function is related to the separation distance, r, by the equation

$$D(r) = 1.89\ \epsilon^{2/3}\ r^{2/3}\ . \tag{20}$$

Taking the square root of equation (20) and changing units of r to nautical miles and $\sqrt{D(r)}$ to knots give

$$\sqrt{D(r)} = \Delta V = 1.52\ \epsilon^{1/3}\ r^{1/3} \tag{21}$$

*The value of 0.5 kn for the maximum current due to internal waves is selected for illustration purposes and is probably toward the upper end of the range of values encountered.

where

ΔV = rms velocity differnece, kn

ϵ = Energy dissipation rate, cm^2/s^3

r = Separation distance, nmi.

In the oceans, ϵ ranges from 10^{-5} cm^2/s^3 to 10^{-1} cm^2/s^3 with 10^{-3} cm^2/s^3 being a "typical" or representative value. Figure 6 gives curves for these three values of ϵ showing how the rms difference between the readings of two ideal speed sensors* separated by a distance r would vary.

UNCERTAINTIES CAUSED BY VERTICAL CURRENT GRADIENTS

The magnitudes of ocean currents characteristically decrease with depth; surface ships and submarines at various depths should not expect to experience the same local water movement. (Current directions also change with depth.)

Selected current velocity profiles appear in figure 7 for three stations in the Atlantic Ocean (selected from 79 current profiles given in the atlas[2]). The solid lines represent average values for the station and the dashed lines represent maximum values. In the presence of gradients such as those shown in figure 7, significant differences can arise in measured speed relative to water even between ships of different drafts, say a carrier and its shallow draft escorts. A submarine operating at a depth different from its target or a submarine changing depth will experience measured speed differences caused solely by the current gradients.

The histogram in figure 8 shows the difference between the average values of surface current and current at 50 meters depth. Based on the 58 average profiles in the North Atlantic, one can expect about 0.4 kn average difference between the magnitudes of surface currents and currents at 50 meters depth. Of course, for certain specific situations, the difference could be as much as 3 to 5 knots.

DISCUSSION

The local water movements, characteristic of the ocean, will cause even a well designed and calibrated water speed sensor to indicate speed readings that do not accurately represent ship motion. For example, a large ship maintaining a

*An ideal water speed sensor is one that senses the true local velocity at a point without perturbing the local flow. A ship's sensor cannot do this because of its finite size and the presence of the ship's hull.

constant revolutions per minute will hold a relatively steady speed over ground.* The water speed sensor measuring the local water velocity at the sensor will measure this steady speed plus the fluctuating velocities because of the orbital velocities in the surface waves. Given sufficient time, these indicated speed perturbations can be reduced to near zero by appropriate data processing and averaging techniques. However, if the ship is maneuvering, the indicated speed must follow the changes in ship's speed to be useful. The time and space scales of these natural oceanic water movements overlap the characteristic time and space scales involved in many ship maneuvers, such as a turn taking several minutes and covering a mile or so of path. The difficulty, then, is how to obtain speed indications that represent ship movements rather than the local water movements.

The magnitude, frequency, and phase of the difference between speed relative to water and speed over ground are dependent on the vehicle parameters, depth of the sensing region below the surface, geographical location, and previous weather conditions. In addition, the difference between the "average" water speed and the instantaneous water speed depends on the "length" of the averaging period or how far one moves through the "local water mass." As an example of spatial effects, consider an acoustic doppler speed sensor that may scatter energy from water several hundred feet deep and thus measure the ship's speed relative to this water. Obviously the effects of surface waves are reduced, but then one must contend with the vertical current gradients (i.e., the change of magnitude and direction of local ocean currents with depth). The measurement and use of "speed relative to the local water mass" requires very careful consideration of what the "local water mass" actually represents.

CONCLUSIONS

- Surface waves cause oscillating horizontal currents with peak magnitudes as high as several knots. The magnitudes decrease with depth with the spectral peaks tending toward lower frequencies (or longer wavelengths). This spectral change may cause difficulties for submarines in selecting a data sampling and processing or averaging scheme that can reduce wave effects and still respond to ship maneuvers.

- The horizontal currents associated with internal waves are proportional to the wave amplitude and phase speed and have magnitudes up to tenths of knots. Since the wave amplitudes do not exceed the upper layer thickness (depth to themocline) and the wave phase speed can be calculated or measured, it should be possible to estimate the upper limit to the horizontal currents associated with internal waves by using local measurements.

*If the ship changes its heading relative to the predominant wave field, the speed over ground may change by several percent for the same revolution per minute because of the changing drag characteristics.

• For vessels of large mass in relatively rapid local water movements, the water motion causes fluctuations in the indicated water speed while the speed over ground is essentially constant. For vessels of lesser mass in slowly varying local water movements, the speed relative to water is very nearly constant while the speed over ground varies as the water motion. These results follow directly from the inertial and drag characteristics of the vessel.

• Turbulence causes random but small (~0.01 to 0.1 kn) fluctuations in the speed readout. The spectral form appropriate to homogeneous, isotropic turbulence may be useful in predicting speed readout variations.

• Local ocean currents tend to decrease in magnitude and shift directions as a function of depth. A submersible and a surface support vessel operating together will almost certainly experience differences between the indicated water speeds of the two vessels even while they maintain zero relative velocity. Typical gradients for the North Atlantic show about 0.4-kn change from the surface to a depth of 50 meters.

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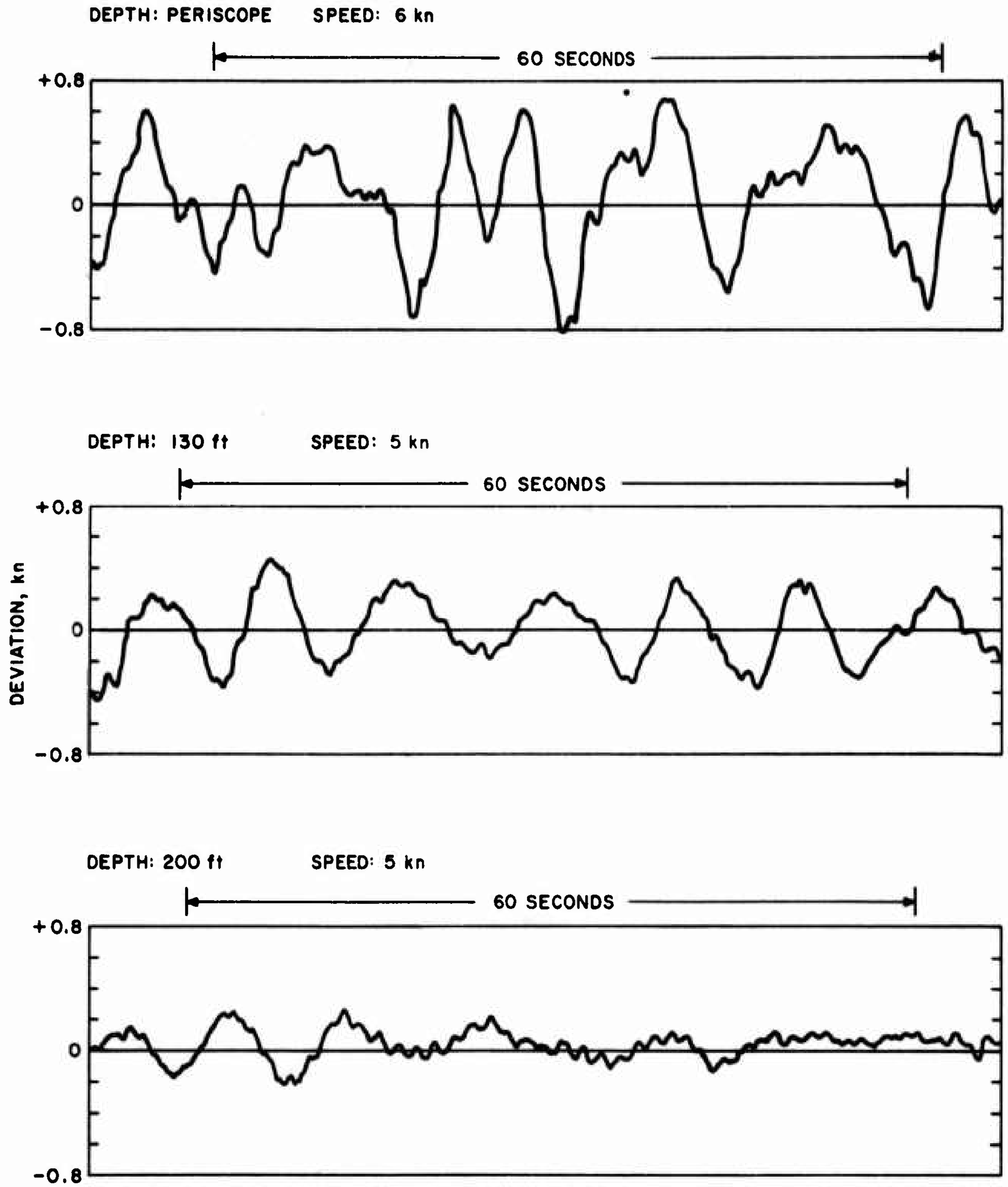


Figure 1
Velocity Variations Recorded on Board Operational Submarine in Sea State 6 at Given Depths and Speeds (Signal Bandpass Approximately 0.08 to 5 Hz)

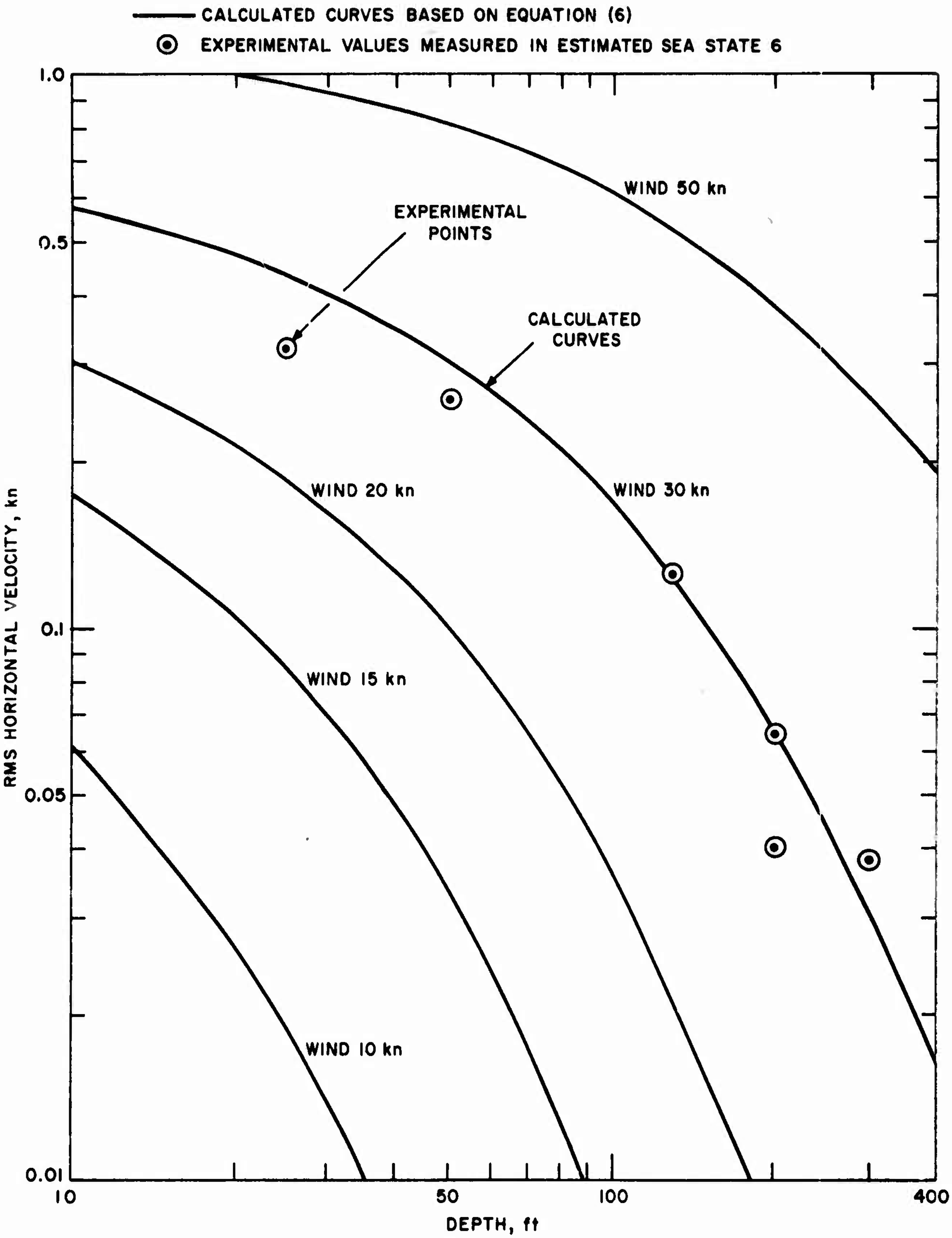
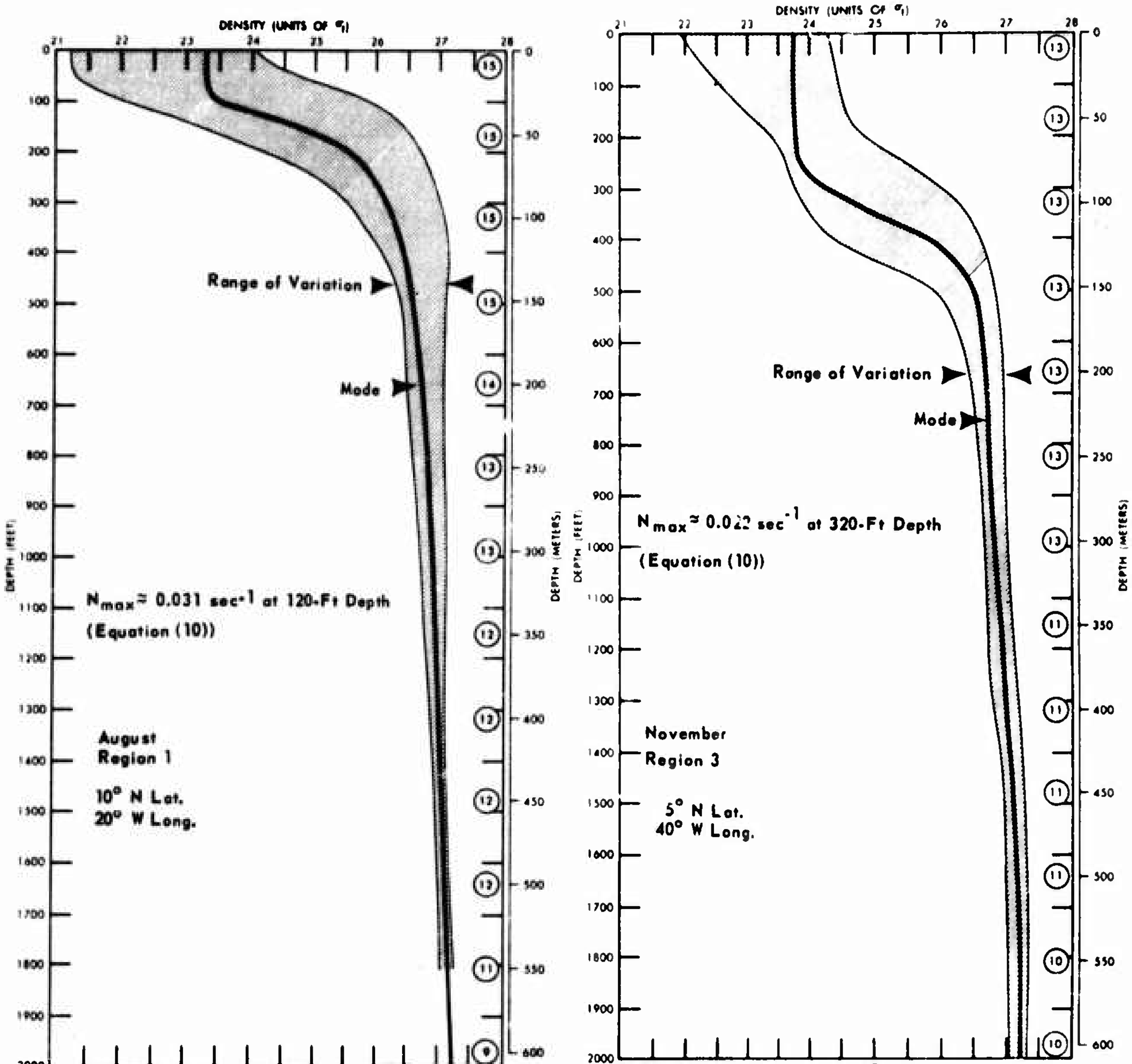


Figure 2
RMS Horizontal Velocity Caused by Wind Generated Surface Waves



From "Physical Properties,"[3] page 173.

From "Physical Properties,"[3] page 195.

Figure 3
Examples of Density Gradients in Atlantic Ocean[3]

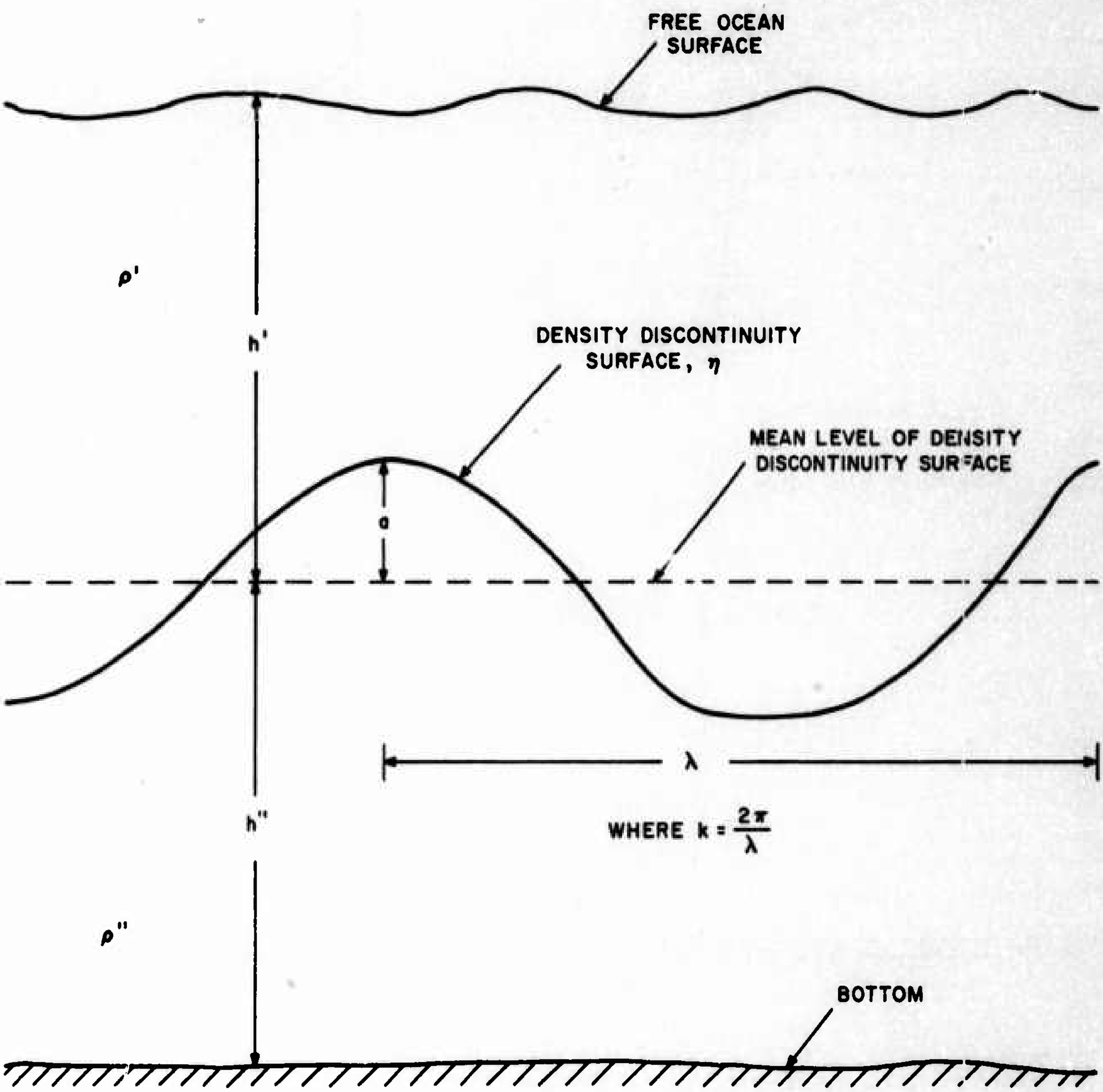


Figure 4
Explanation of Symbols Used in Internal Wave Equations

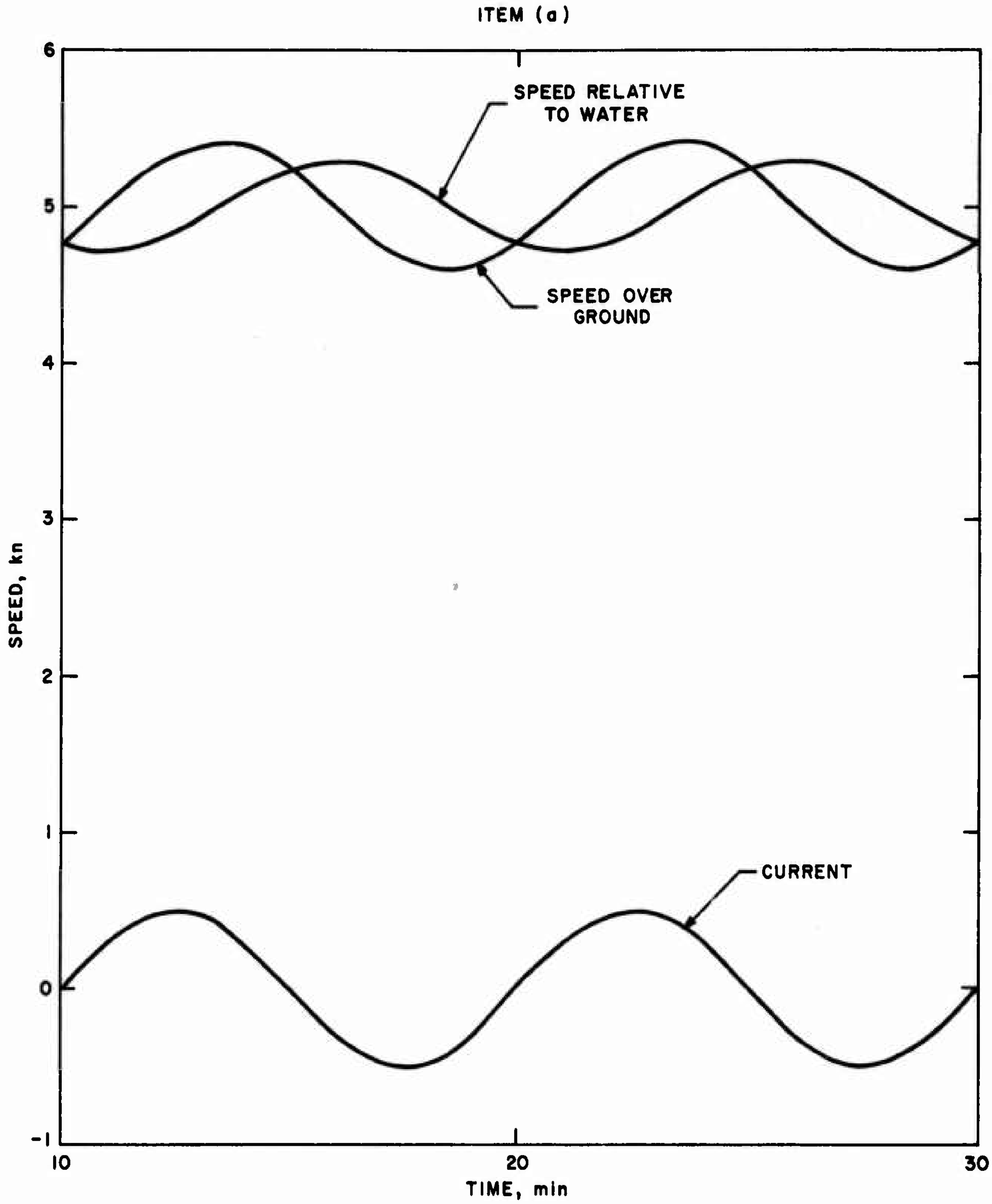


Figure 5
Variations of Speed of Submersible in Oscillatory Current Due to Internal Waves

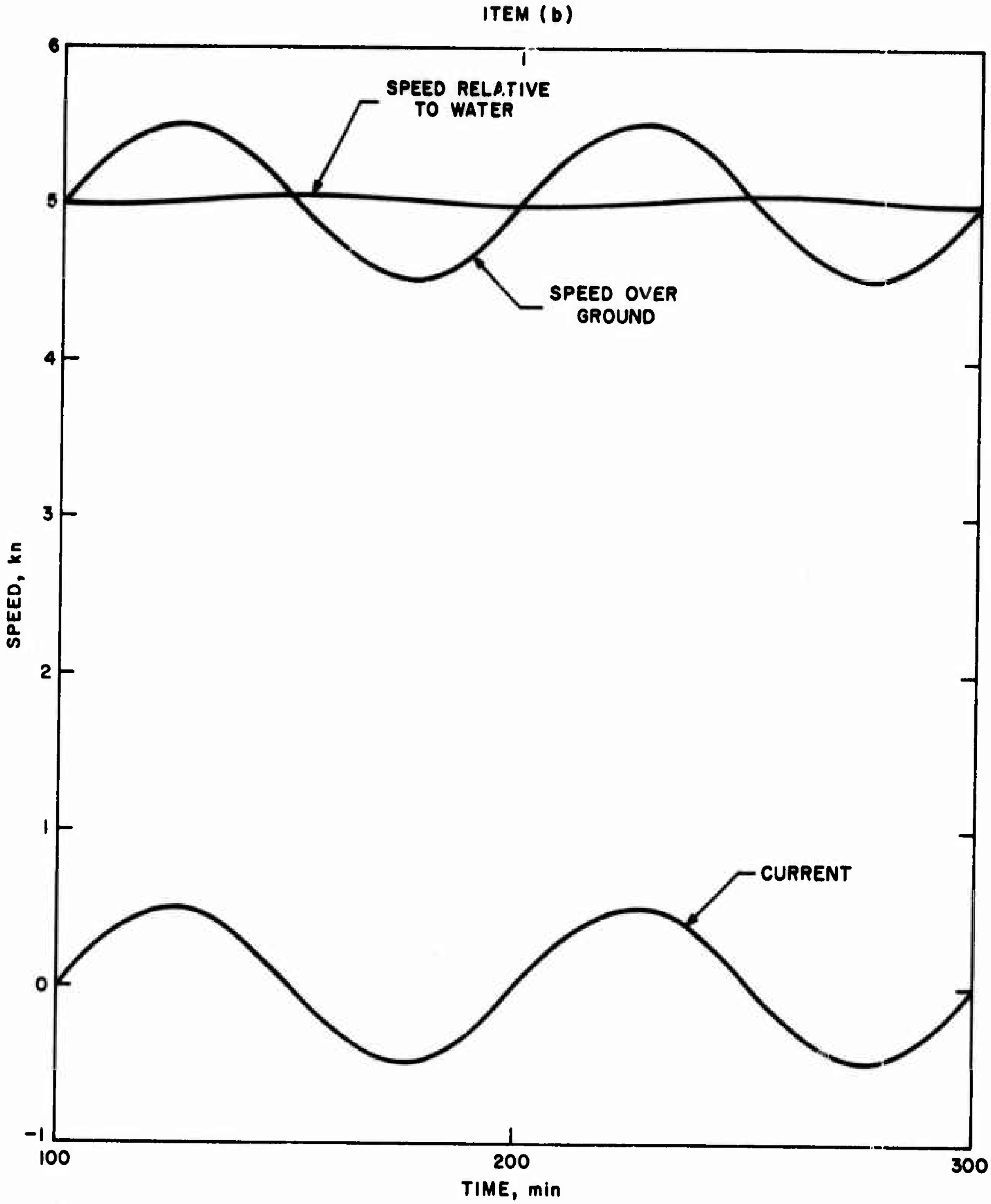


Figure 5 (Cont)

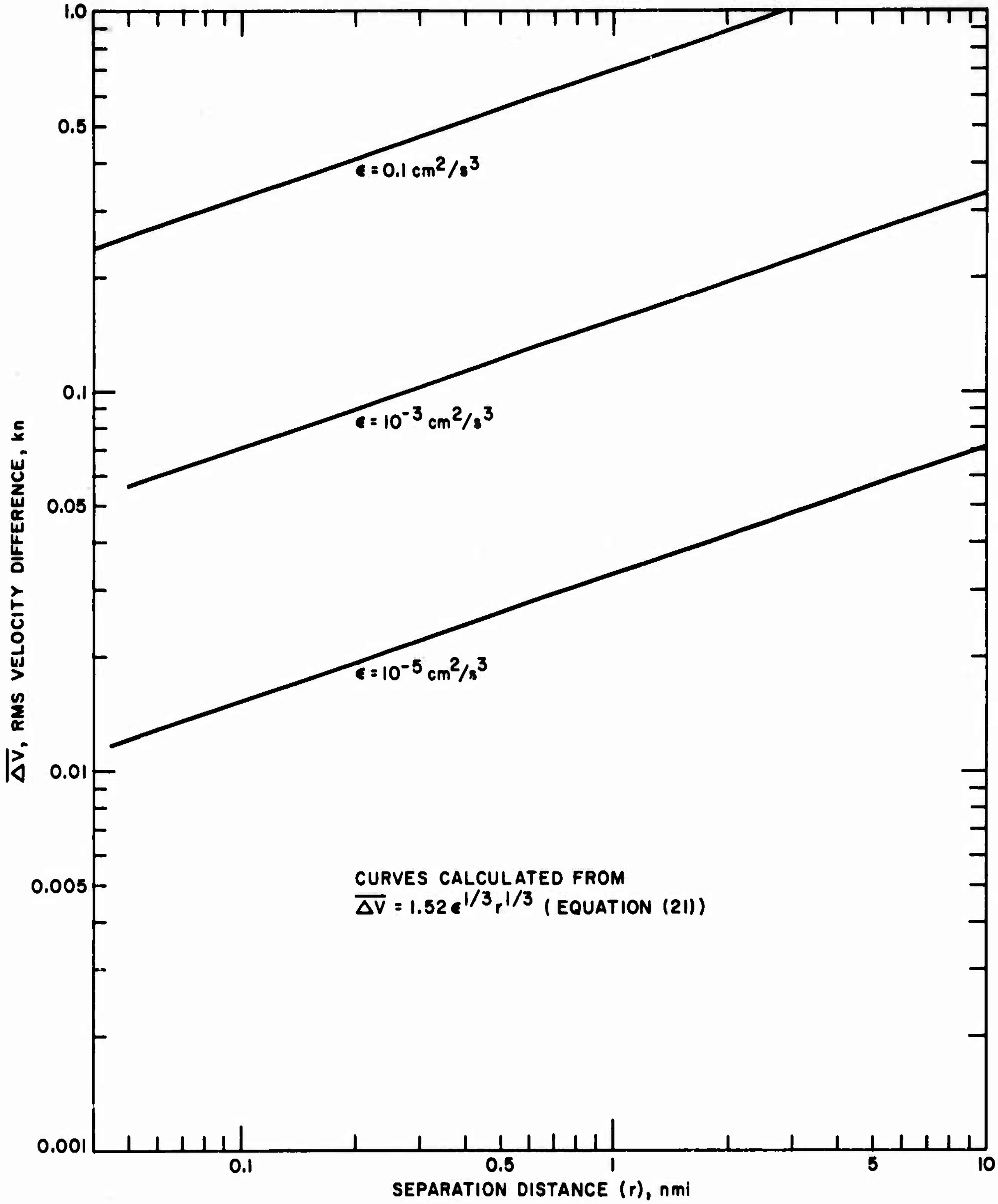
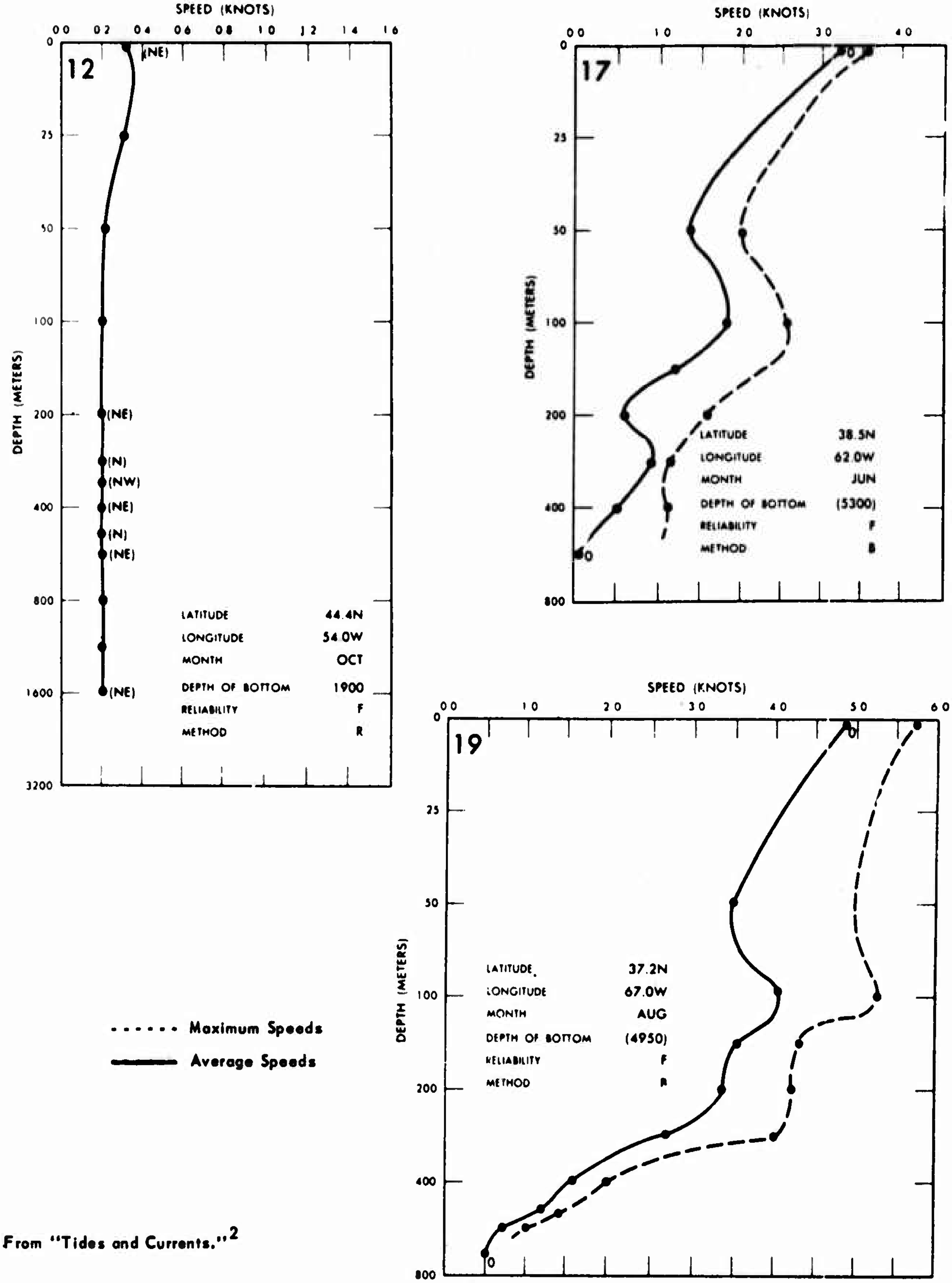


Figure 6
RMS Difference Between Water Velocity at Two Points Separated by a Distance r in an Isotropic, Homogeneous Turbulence Field Characterized by Dissipation Rate, ϵ.



From "Tides and Currents."[2]

Figure 7
Vertical Velocity Profiles for Selected Atlantic Stations

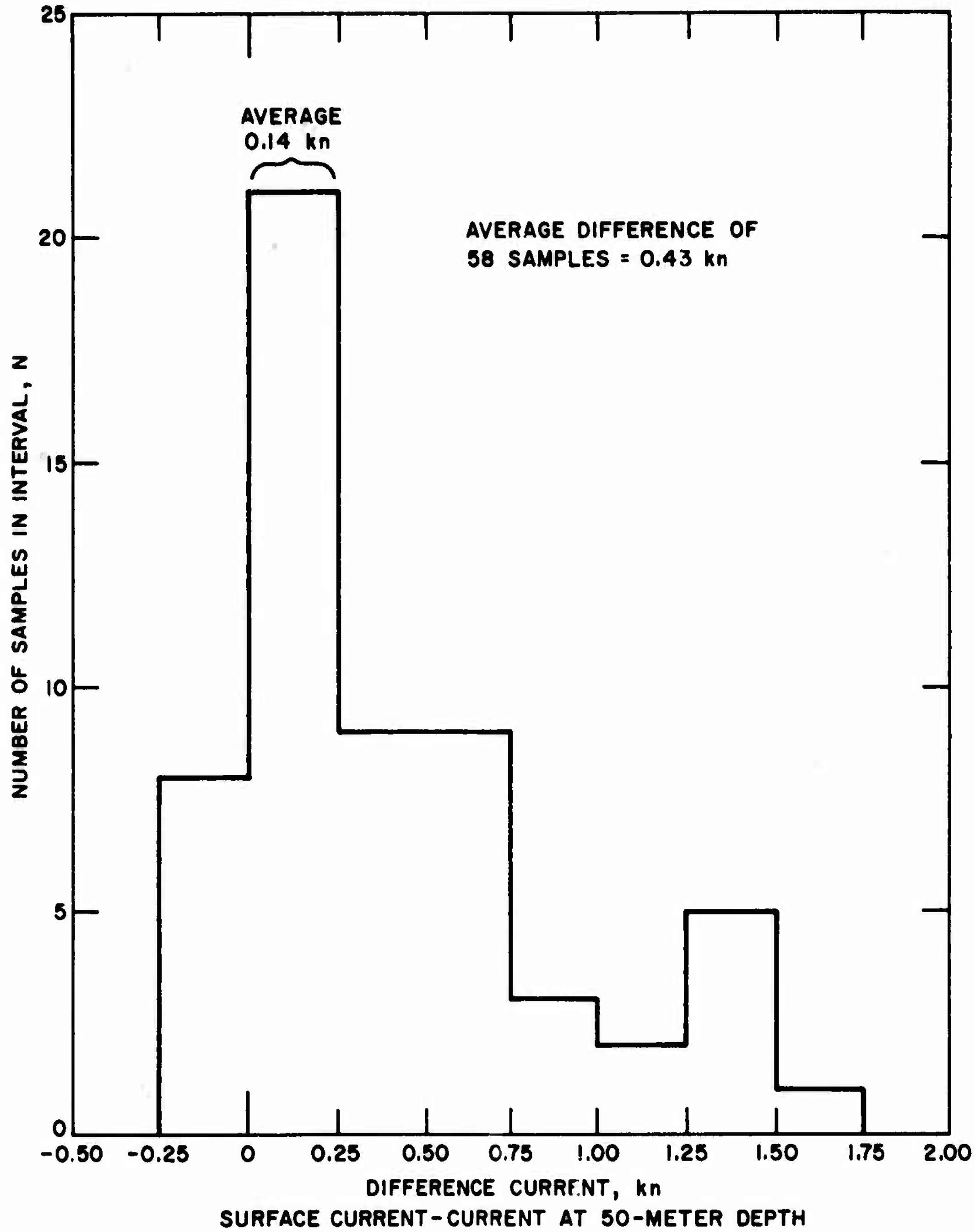


Figure 8
Histogram of Difference between Current Magnitude at Surface and 50-meter Depth for 58 Stations in Atlantic Ocean.

APPENDIX A

SPEED-MEASUREMENT EFFECTS CAUSED BY OSCILLATORY CURRENTS

The oscillatory currents in an internal wave-field will cause a variation of indication from a speed sensor that is measuring vehicle speed relative to the water. The drag forces on the submersible vehicle depend on its velocity relative to the water, but the acceleration forces depend on velocity changes relative to the ground. The oscillating horizontal currents of internal waves affect both speed relative to water and speed over the ground.

Consider a submersible of mass M maintaining a constant thrust force T_o and moving in a region of oscillatory horizontal currents v given by

$$v = v_o \sin \omega t$$

where:

v_o = the peak current value

ω = the encounter frequency seen by an observer on the submersible

t = time.

Assuming the drag force is proportional to the square of the speed relative to the water, the equation of motion is

$$M \frac{dV}{dt} + C_d(V-v)^2 + T_o = 0$$

where:

M = mass of submersible

V = speed of submersible relative to ground

C_d = drag force coefficient

v = current relative to ground

T_o = constant thrust force.

To obtain a solution, note that if $v = 0$, the constant thrust T_o balances the drag at steady speed V_o such that

$$C_d V_o^2 + T_o = 0 \ .$$

The speed over ground V can be expressed as the steady speed V_o plus a perturbation u,

$$V = V_o + u \ .$$

Substituting this into the governing differential equation and rearranging terms gives

$$M \frac{du}{dt} + C_d \left[2V_o(u-v) + (u-v)^2\right] = 0 \ .$$

Assuming $u-v \ll V_o$, the equation simplifies to

$$M \frac{du}{dt} + 2C_d V_o (u-v) = 0 \ .$$

Substituting for v and rearranging now gives

$$\frac{du}{dt} + \frac{2C_d V_o}{M} u = \frac{2C_d V_o}{M} v_o \sin \omega t.$$

The solution to this equation is

$$u = v_o \cos \varphi \sin (\omega t + \varphi)$$

where

$$\tan \varphi = - \frac{\omega M}{2V_o C_d} \ .$$

The speed over the ground is given by $V = V_o + u$ or

$$V = V_o + v_o \cos \varphi \sin (\omega t + \varphi) \ .$$

The indicated speed (or speed relative to the water) is given by $V_I = V - v$ or

$$V_I = V_o + v_o \sin \varphi \cos (\omega t + \varphi) \ .$$